

On generalized Born–Infeld electrodynamics

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Abstract

The generalized Born–Infeld electrodynamics with two parameters is investigated. In this model the propagation of a linearly polarized laser beam in the external transverse magnetic field is considered. It was shown that there is the effect of vacuum birefringence, and we evaluate induced ellipticity. The upper bounds on the combination of parameters introduced from the experimental data of BRST and PVLAS Collaborations were obtained. When two parameters are equal to each other, we arrive at Born–Infeld electrodynamics and the effect of vacuum birefringence vanishes. We find the canonical and symmetrical Belinfante energy-momentum tensors. The trace of the energy-momentum tensor is not zero and the dilatation symmetry is broken. The four-divergence of the dilatation current is equal to the trace of the Belinfante energy-momentum tensor and is proportional to the parameter (with the dimension of the field strength) of the model. The dual symmetry is also broken in the model considered.

1 Introduction

A renewal of interest in the Born–Infeld (BI) theory is due to the development of string theories. Thus, the low energy D-brane dynamics is governed by a BI type action [1]. Firstly, a non-linear BI electrodynamics was formulated [2], [3] to have a finite electromagnetic energy of a point charge contrarily to Maxwell’s electrodynamics. The fundamental constant B introduced in BI electrodynamics, with the dimension of the field strength, characterizes the upper bound for the possible electromagnetic fields. When the B approaches to infinity the BI theory converts to Maxwell’s electrodynamics. Some aspects of wave propagation in Born-Infeld electrodynamics were investigated in [4], [5], [6], [7].

In this paper, we formulate and investigate the generalized BI electrodynamics with two parameters. If two parameters introduced are equal, we come to BI electrodynamics.

The paper is organized as follows. In section 2, the Lagrangian of the model is motivated. The field equations in generalized BI electrodynamics are formulated in section 3. We investigate the propagation of a linearly polarized laser beam in the external transverse magnetic field and evaluate induced ellipticity in section 4. The upper bounds on the combination of parameters β and γ introduced are obtained from the experimental data of BRST and PVLAS Collaborations. In section 5., we find the canonical and symmetrical Belinfante energy-momentum tensors, the dilatation current, and its non-zero divergence. The results are discussed in section 6.

The Heaviside–Lorentz system with $\hbar = c = 1$ is chosen, and Euclidian metric is used, $x_\mu = (x_m, ix_0)$, x_0 is a time. Greek letters range from 1 to 4 and Latin letters range from 1 to 3.

2 The model

Let us consider the Born–Infeld Lagrangian density [2], [3]

$$\mathcal{L}^{\mathcal{B}-\mathcal{I}} = B^2 \left(1 - \sqrt{1 + 2B^{-2}S - B^{-4}P^2} \right), \quad (1)$$

where two Lorentz-invariants are given by

$$S = \frac{1}{4}F_{\mu\nu}^2, \quad P = \frac{1}{4}F_{\mu\nu}\tilde{F}_{\mu\nu}, \quad (2)$$

$F_{\mu\nu}$ is the field strength, and $\tilde{F}_{\mu\nu} = (1/2)\varepsilon_{\mu\nu\alpha\beta}F_{\alpha\beta}$ is its dual tensor ($\varepsilon_{1234} = -i$). The parameter B characterizes the upper bound on the possible field strength. The Lagrangian density (1) is converted to the Maxwell’s Lagrangian density when the fields are weak compared to the parameter B . Thus, for small values of $B^{-2}S$, $B^{-4}P^2$, in leading order, equation (1) becomes

$$\mathcal{L}^{\mathcal{B}-\mathcal{I}} \simeq -S + \frac{1}{2B^2}S^2 + \frac{1}{2B^2}P^2. \quad (3)$$

At the same time quantum one-loop corrections in QED lead to the Heisenberg–Euler Lagrangian density of self-interaction [10], [11], [18]

$$\mathcal{L}^{\mathcal{H}-\mathcal{E}} = 4aS^2 + bP^2, \quad (4)$$

where

$$a = \frac{2\alpha^2}{45m_e^4}, \quad b = \frac{14\alpha^2}{45m_e^4}, \quad (5)$$

and $\alpha = e^2/(4\pi) \simeq 1/137$ is the fine structure constant, and the m_e is the electron mass. We imply that in quantum BI theory the same quantum corrections appear as in QED at least in the perturbation theory because at large value of B the BI Lagrangian becomes the classical electrodynamics Lagrangian. As a result, we may consider the approximate BI Lagrangian density (3) with quantum corrections (4) that leads to the effective Lagrangian density:

$$\mathcal{L}_{eff} = \mathcal{L}^{B-\mathcal{I}} + \mathcal{L}^{\mathcal{H}-\mathcal{E}} \simeq -S + \frac{1}{2\beta^2}S^2 + \frac{1}{2\gamma^2}P^2, \quad (6)$$

where

$$\frac{1}{\beta^2} = \frac{1}{B^2} + 8a, \quad \frac{1}{\gamma^2} = \frac{1}{B^2} + 2b. \quad (7)$$

The nonlinear model of the type (6) appears due to the vacuum polarization of arbitrary spin particles [8]. The propagation of a linearly polarized laser light in the external transverse magnetic field, in the effective theory (6), was investigated in [9]. If $\beta \neq \gamma$, the model (6) leads to birefringence: the phase velocities of light depend on polarizations in the external electromagnetic fields. There is no birefringence in BI electrodynamics. The Heisenberg–Euler Lagrangian [10], [11], [18] corresponding to the one-loop corrections to classical electrodynamics follows form (6), (7) at $B \rightarrow \infty$. Therefore, the case $\beta \neq \gamma$ is natural due to quantum corrections of Maxwell’s electrodynamics. The case of the finiteness of β and vanishing γ^{-2} ($\gamma \rightarrow \infty$) in equation (6) (ignoring relations (7)), was considered [13] in the framework of the Kaluza–Klein theory (in five dimensions) with the additional Gauss–Bonnet term. Now we can introduce the generalized BI Lagrangian density

$$\mathcal{L} = \beta^2 \left(1 - \sqrt{1 + 2\beta^{-2}S - \beta^{-2}\gamma^{-2}P^2} \right), \quad (8)$$

which leads to equation (6) at small values of $\beta^{-2}S$, $\beta^{-2}\gamma^{-2}P^2$. Thus, the model with Lagrangian density (8) with two different parameters β and γ can be considered as the natural generalization of BI electrodynamics. We treat this non-linear model as the effective electrodynamics theory for strong electromagnetic fields. This model allows us to consider particular cases. Of course our discussion of motivating Lagrangian density (8) is not strict

enough and is heuristic. One may consider the model based on equation (8) as a convenient parametrization which allows us to analyze different models discussed in literature. Anyway, a model (8) in our opinion has a definite theoretical interest.

3 The field equations

Introducing, as usual, the four-vector potential A_μ : $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, using Eq.(8), we find the equations of motion from the Euler-Lagrange equations ($\partial_\mu \partial \mathcal{L} / \partial (\partial_\mu A_\nu) - \partial \mathcal{L} / \partial A_\nu = 0$):

$$\partial_\mu \left[\frac{1}{\mathcal{R}} \left(F_{\mu\nu} - \frac{P}{\gamma^2} \tilde{F}_{\mu\nu} \right) \right] = 0, \quad (9)$$

where

$$\mathcal{R} = \sqrt{1 + 2\beta^{-2}S - \beta^{-2}\gamma^{-2}P^2}. \quad (10)$$

Other equations follow from the Bianchi identity:

$$\partial_\mu \tilde{F}_{\mu\nu} = 0. \quad (11)$$

Equations (9),(11) may be cast into the form of nonlinear Maxwell's equations. Introducing the electric displacement field $\mathbf{D} = \partial \mathcal{L} / \partial \mathbf{E}$, one obtains from Eq.(8)

$$\mathbf{D} = \frac{1}{\mathcal{R}} \left(\mathbf{E} + \frac{1}{\gamma^2} \mathbf{B}(\mathbf{E} \cdot \mathbf{B}) \right). \quad (12)$$

The magnetic field is given by $\mathbf{H} = -\partial \mathcal{L} / \partial \mathbf{B}$:

$$\mathbf{H} = \frac{1}{\mathcal{R}} \left(\mathbf{B} - \frac{1}{\gamma^2} \mathbf{E}(\mathbf{E} \cdot \mathbf{B}) \right), \quad (13)$$

where $E_j = iF_{j4}$, $B_j = (1/2)\varepsilon_{jik}F_{ik}$ ($\varepsilon_{123} = 1$). Eq.(9) can be represented in the terms of fields \mathbf{D} and \mathbf{H} :

$$\nabla \cdot \mathbf{D} = 0, \quad \frac{\partial \mathbf{D}}{\partial t} - \nabla \times \mathbf{H} = 0. \quad (14)$$

The second pair of Maxwell's equation follows from equation (11):

$$\nabla \cdot \mathbf{B} = 0, \quad \frac{\partial \mathbf{B}}{\partial t} + \nabla \times \mathbf{E} = 0. \quad (15)$$

From equations (12),(13), we obtain the electric permittivity ε_{ik} and the inverse magnetic permeability $(\mu)_{ik}^{-1}$ tensors of the vacuum

$$\varepsilon_{ik} = \frac{1}{\mathcal{R}} \left(\delta_{ik} + \frac{1}{\gamma^2} B_i B_k \right), \quad (\mu)_{ik}^{-1} = \frac{1}{\mathcal{R}} \left(\delta_{ik} - \frac{1}{\gamma^2} E_i E_k \right), \quad (16)$$

with the connections

$$D_i = \varepsilon_{ik} E_k, \quad B_i = \mu_{ik} H_k. \quad (17)$$

Thus, the generalized BI electrodynamics equations with two parameters were rewritten in the form of Maxwell's equations (14),(15) with the nonlinear links (17) (because of (16)).

Equations (16) show that the vacuum possesses the complicated anisotropic properties. Therefore, it is useful to find eigenvalues of the electric permittivity and magnetic permeability tensors. From (16), one finds the “minimal” polynomials of matrices $\varepsilon = (\varepsilon_{ik})$ and $\mu^{-1} = ((\mu^{-1})_{ik})$:

$$\begin{aligned} \left[\varepsilon - \frac{1}{\mathcal{R}} \left(1 + \frac{\mathbf{B}^2}{\gamma^2} \right) \right] \left(\varepsilon - \frac{1}{\mathcal{R}} \right) &= 0, \\ \left[\mu^{-1} - \frac{1}{\mathcal{R}} \left(1 - \frac{\mathbf{E}^2}{\gamma^2} \right) \right] \left(\mu^{-1} - \frac{1}{\mathcal{R}} \right) &= 0. \end{aligned} \quad (18)$$

Equations (18) allow us to obtain eigenvalues $\lambda_{1,2}(\varepsilon, \mu^{-1})$ and inverse matrices ε^{-1} , μ :

$$\begin{aligned} \lambda_1(\varepsilon) &= \frac{1}{\mathcal{R}} \left(1 + \frac{\mathbf{B}^2}{\gamma^2} \right), \quad \lambda_2(\varepsilon) = \frac{1}{\mathcal{R}}, \\ \lambda_1(\mu^{-1}) &= \frac{1}{\mathcal{R}} \left(1 - \frac{\mathbf{E}^2}{\gamma^2} \right), \quad \lambda_2(\mu^{-1}) = \frac{1}{\mathcal{R}}, \end{aligned} \quad (19)$$

$$\begin{aligned} (\varepsilon^{-1})_{ik} &= \mathcal{R} \left(\delta_{ik} - \frac{\gamma^{-2} B_i B_k}{1 + \gamma^{-2} \mathbf{B}^2} \right), \\ \mu_{ik} &= \mathcal{R} \left(\delta_{ik} + \frac{\gamma^{-2} E_i E_k}{1 - \gamma^{-2} \mathbf{E}^2} \right). \end{aligned} \quad (20)$$

As in BI electrostatics (at $\mathbf{B} = \mathbf{H} = 0$), we write one of equations (14) with the point source

$$\nabla \cdot \mathbf{D}_0 = e\delta(\mathbf{r}) \quad (21)$$

possessing the solution

$$\mathbf{D}_0 = \frac{e}{4\pi r^3} \mathbf{r}. \quad (22)$$

Then from (16),(17), one obtains the electric field

$$\mathbf{E}_0 = \frac{\mathbf{D}_0}{\sqrt{1 + \beta^{-2} D_0^2}} = \frac{e \mathbf{r}}{4\pi r \sqrt{r^4 + r_0^4}}, \quad (23)$$

where the elementary length is $r_0 = \sqrt{|e|/4\pi\beta}$. The electric field of a point-like charge is not singular resulting to the finiteness of energy contrarily to linear electrodynamics. Thus, the parameter γ introduced does not enter electrostatics equations and all equations remain the same as in BI electrodynamics with the replacement $B \rightarrow \beta$. It follows from equation (23) that the maximum field strength is $E_{max} = \lim_{r \rightarrow 0} E_0 = \beta$. The lower bound on the BI parameter B [14] is given by $B \geq 10^{20}$ V/m.

From equations (12),(13), we obtain the relation

$$\mathbf{D} \cdot \mathbf{H} = \mathbf{E} \cdot \mathbf{B} \frac{1 + 2\gamma^{-2}S - \gamma^{-4}P^2}{1 + 2\beta^{-2}S - \beta^{-2}\gamma^{-2}P^2}. \quad (24)$$

According to the criterion of [15], the nonlinear electrodynamics possesses the duality symmetry if $\mathbf{D} \cdot \mathbf{H} = \mathbf{E} \cdot \mathbf{B}$. It follows from (24) that the duality symmetry is broken in the model considered if $\beta \neq \gamma$. In BI electrodynamics $\beta = \gamma \equiv B$ and the duality symmetry is recovered. In QED due to quantum corrections $4a \neq b$ ($\beta \neq \gamma$, see equations (5),(7)), and the duality symmetry is broken.

4 Propagation of waves in the background magnetic field

Now we analyze the propagation of the plane electromagnetic wave (\mathbf{e}, \mathbf{b}) traveling in z -direction and perpendicular to the external constant and uniform magnetic induction field $\mathbf{\overline{B}} = (\overline{B}, 0, 0)$. We consider the Lagrangian (8) without the approximation of weakness of external fields. The electromagnetic fields are the sum of the light field and the background magnetic field, $\mathbf{E} = \mathbf{e}$, $\mathbf{B} = \mathbf{b} + \mathbf{\overline{B}}$. It is implied that the wave fields are much weaker than external magnetic induction field. Replacing the decomposition of fields in

equation (8), and neglecting the higher order in light fields, one obtains the Lagrangian

$$\mathcal{L}(\mathbf{e}, \mathbf{b} + \overline{\mathbf{B}}) = \beta^2 \left(1 - \sqrt{1 + \frac{(\mathbf{b} + \overline{\mathbf{B}})^2 - \mathbf{e}^2}{\beta^2}} - \frac{(\mathbf{e} \cdot \mathbf{B})^2}{\beta^2 \gamma^2} \right). \quad (25)$$

We left in (25) only quadratic terms in \mathbf{e} and \mathbf{b} . One can find from equation (25) the displacement and induction magnetic vectors in the background fields

$$d_i = \frac{\partial \mathcal{L}}{\partial e_i} = \frac{1}{\kappa} \left(\delta_{ij} + \frac{1}{\gamma^2} \overline{B}_i \overline{B}_j \right) e_j, \quad (26)$$

$$h_i = -\frac{\partial \mathcal{L}}{\partial b_i} = \frac{1}{\kappa} \left(\delta_{ij} - \frac{1}{\kappa^2 \beta^2} \overline{B}_i \overline{B}_j \right) b_j, \quad (27)$$

where

$$\kappa = \sqrt{1 + \frac{\overline{\mathbf{B}}^2}{\beta^2}}, \quad (28)$$

and only linear terms in \mathbf{e} and \mathbf{b} are left in (26),(27). It should be noted that we do not imply the smallness of the background magnetic fields. Thus, the unitless parameters $\overline{\mathbf{B}}^2/\beta^2$, $\overline{\mathbf{B}}^2/\gamma^2$ can be arbitrary. From equations (26),(27), using the relations $d_i = \varepsilon_{ij} e_j$, $h_i = (\mu^{-1})_{ij} b_j$, we obtain the polarization tensors in the external magnetic field

$$\varepsilon_{ij} = \frac{1}{\kappa} \left(\delta_{ij} + \frac{1}{\gamma^2} \overline{B}_i \overline{B}_j \right), \quad (\mu^{-1})_{ij} = \frac{1}{\kappa} \left(\delta_{ij} - \frac{1}{\kappa^2 \beta^2} \overline{B}_i \overline{B}_j \right). \quad (29)$$

From Maxwell equations, one finds the equation for the electric wave field as follows [9]:

$$\left[\mathbf{k}^2 (\mu^{-1})_{bi} + k_a (\mu^{-1})_{al} k_l \delta_{ib} - \mathbf{k}^2 (\mu^{-1})_{aa} \delta_{ib} - k_l (\mu^{-1})_{bl} k_i + \omega^2 \varepsilon_{ib} \right] e_b = 0. \quad (30)$$

It follows from homogeneous equation (30) that nontrivial solutions exist when the determinant of the matrix is zero. Replacing (29) into (30), we obtain the corresponding matrix

$$\Lambda_{ij} = \left(1 - n^2 + n^2 \frac{\overline{\mathbf{B}}^2}{\kappa^2 \beta^2} \right) \delta_{ij} + \left(\frac{1}{\gamma^2} - \frac{n^2}{\kappa^2 \beta^2} \right) \overline{B}_i \overline{B}_j, \quad (31)$$

where the index of refraction is $n = k/\omega$. With the help of the method of [9], one finds the eigenvalues of the matrix (31):

$$\lambda_1 = 1 - n^2 \left(1 - \frac{\overline{\mathbf{B}}^2}{\kappa^2 \beta^2} \right), \quad \lambda_2 = 1 - n^2 + \frac{\overline{\mathbf{B}}^2}{\gamma^2}. \quad (32)$$

From equation (32), with the help of (28), we obtain the indexes of refraction for two modes corresponding to $\lambda_1 = 0$ and $\lambda_2 = 0$:

$$n_\perp = \sqrt{1 + \frac{\overline{\mathbf{B}}^2}{\beta^2}}, \quad n_\parallel = \sqrt{1 + \frac{\overline{\mathbf{B}}^2}{\gamma^2}}. \quad (33)$$

Thus, the electromagnetic waves with different polarizations have different velocities $v_\perp = n_\perp^{-1}$, $v_\parallel = n_\parallel^{-1}$, and there is the effect of vacuum birefringence if $\beta \neq \gamma$. Let the polarization vector at $z = 0$ is $\mathbf{e}|_{z=0} = E_0(\cos \theta, \sin \theta) \exp(-i\omega t)$, where θ is the angle between the polarization vector \mathbf{e} and the external magnetic induction field $\overline{\mathbf{B}}$. Then the linearly polarized wave traveling the distance L becomes the elliptically polarized wave. According to [9], ellipticity (the ratio of minor to major axis of the ellipse) is given by

$$\Psi = \frac{1}{2} (n_\perp - n_\parallel) \omega L \sin 2\theta, \quad (34)$$

where $\omega = 2\pi/\lambda$, and λ is a wave length. In BI electrodynamics $\beta = \gamma$ ($n_\perp = n_\parallel$) and vacuum birefringence vanishes.

In the case of the smallness of the parameters $\overline{\mathbf{B}}^2/\beta^2$, $\overline{\mathbf{B}}^2/\gamma^2$, the indexes of refraction (33) become

$$n_\perp \simeq 1 + \frac{1}{2\beta^2} \overline{\mathbf{B}}^2, \quad n_\parallel \simeq 1 + \frac{1}{2\gamma^2} \overline{\mathbf{B}}^2. \quad (35)$$

Replacing equation (35) into (34), one obtains the result of [9] corresponding to the Lagrangian (6). Let us estimate the upper bounds on the value

$$\Delta = \frac{1}{\beta^2} - \frac{1}{\gamma^2} \quad (36)$$

with the help of polarization data for ellipticity Ψ of BRST [16] and PVLAS [17] Collaborations. The experimental apparatus of BRST Collaboration consisted of a magnetic-field region and the ellipsometer, where the resulting

polarization change was measured. The magnetic field was supplied by two superconducting dipole magnets, and the length of the two magnets was 8.8 m ($1 \text{ m} = 5.1 \times 10^6 \text{ eV}^{-1}$). They studied the propagation of a laser beam with the wavelength $\lambda = 514.5 \text{ nm}$ through a transverse magnetic field of 3.25 T ($\overline{B} = 3.25 \text{ T}$, $1 \text{ T} = 195.5 \text{ eV}^2$ in Heaviside–Lorentz’s units), and searched for light scalar and/or pseudoscalar particles that couple to two photons. The input polarizer was set at 45° ($\theta = 45^\circ$) to the direction of the magnetic field. The number of reflections was N so that NL is the total optical path length. From BRST [16] data, one obtains

$$N = 578, \quad \Psi = 40 \text{ nrad}, \quad \Delta = 6.38 \times 10^{-24} \text{ eV}^{-4},$$

$$N = 34, \quad \Psi = 1.6 \text{ nrad}, \quad \Delta = 4.34 \times 10^{-24} \text{ eV}^{-4}.$$

In the PVLAS experiment, the wavelength of $\lambda = 1064 \text{ nm}$ was used with the magnetic field strengths of $\overline{B} = 2.3 \text{ T}$. The setup consisted of a sensitive ellipsometer detected changes in the polarization state of light propagating through a $L = 1 \text{ m}$ long magnetic field region in vacuum. It was based on a high finesse Fabry-Perot cavity and a superconducting rotating dipole magnet. The results from measurements did not confirm the presence of a rotation signal and excluded an ellipticity signal at 2.3 T. To check a presence of fringe field effects, two measurements in vacuum were performed with the apparatus in the configuration at $\overline{B} = 2.3 \text{ T}$ field (there was no a fringe field), and at a 5 T field (there was a fringe field). Thus, at the 2.3 T measurements, no visible signal peak was observed both in rotation and in ellipticity. It was concluded in [17] that the rotation measurements at the field intensity of 5 T indicated that the rotation signal reported in previous publications was due to an instrumental artifact. It should be noted that the rotation of the magnetic field in the PVLAS experiment does not effect on the value of vacuum birefringence within QED calculations [18], [19]. The limiting observed background value for ellipticity is $\Psi < 0.31 \text{ prad/pass}$ with $N = 45000$ passes in the interaction region. Using the value $\overline{B} = 2.3 \text{ T}$, $L = 1 \text{ m}$, and $\theta = \pi/4$ of PVLAS setup, we obtain the upper bound on the value of the parameter Δ (36):

$$\Delta < 1 \times 10^{-24} \text{ eV}^{-4}.$$

We note that new results exclude the particle interpretation of the previous PVLAS results as due to a spin zero boson.

5 The energy-momentum tensor and dilation current

With the help of the Lagrangian (8), we find the conserved canonical energy-momentum tensor $T_{\mu\nu}^c = (\partial_\nu A_\alpha) \partial \mathcal{L} / \partial (\partial_\mu A_\alpha) - \delta_{\mu\nu} \mathcal{L}$:

$$T_{\mu\nu}^c = (\partial_\nu A_\alpha) \frac{1}{\mathcal{R}} \left(\frac{P}{\gamma^2} \tilde{F}_{\mu\alpha} - F_{\mu\alpha} \right) - \delta_{\mu\nu} \mathcal{L}, \quad (37)$$

and $\partial_\mu T_{\mu\nu}^c = 0$. The tensor (37) is not symmetric and gauge-invariant tensor. We can obtain the symmetric Belinfante tensor using the relation [20]:

$$T_{\mu\nu}^B = T_{\mu\nu}^c + \partial_\beta X_{\beta\mu\nu}, \quad (38)$$

where

$$X_{\beta\mu\nu} = \frac{1}{2} \left[\Pi_{\beta\sigma} (\Sigma_{\mu\nu})_{\sigma\rho} - \Pi_{\mu\sigma} (\Sigma_{\beta\nu})_{\sigma\rho} - \Pi_{\nu\sigma} (\Sigma_{\beta\mu})_{\sigma\rho} \right] A_\rho, \quad (39)$$

$$\Pi_{\mu\sigma} = \frac{\partial \mathcal{L}}{\partial (\partial_\mu A_\sigma)} = \frac{1}{\mathcal{R}} \left(\frac{P}{\gamma^2} \tilde{F}_{\mu\sigma} - F_{\mu\sigma} \right). \quad (40)$$

The tensor $X_{\beta\mu\nu}$ is antisymmetric in indexes β and μ , and therefore $\partial_\mu \partial_\beta X_{\beta\mu\nu} = 0$. As a result $\partial_\mu T_{\mu\nu}^B = \partial_\mu T_{\mu\nu}^c = 0$ and the symmetric Belinfante tensor is also conserved. The generators of the Lorentz transformations $\Sigma_{\mu\alpha}$ have the matrix elements:

$$(\Sigma_{\mu\alpha})_{\sigma\rho} = \delta_{\mu\sigma} \delta_{\alpha\rho} - \delta_{\alpha\sigma} \delta_{\mu\rho}. \quad (41)$$

With the help of equations (39),(40), one finds

$$\partial_\beta X_{\beta\mu\nu} = \Pi_{\beta\mu} \partial_\beta A_\nu. \quad (42)$$

It follows from equation of motion (9) that $\partial_\mu \Pi_{\mu\nu} = 0$, and one can verify that the equation $\partial_\mu \partial_\beta X_{\beta\mu\nu} = 0$ holds. Using equations (40),(42), the conserved Belinfante tensor (38) becomes symmetric and gauge-invariant tensor:

$$T_{\mu\nu}^B = \frac{F_{\nu\alpha}}{\mathcal{R}} \left(\frac{P}{\gamma^2} \tilde{F}_{\mu\alpha} - F_{\mu\alpha} \right) - \delta_{\mu\nu} \mathcal{L}. \quad (43)$$

The symmetric energy-momentum tensor (43) also can be obtained by varying the action $S = \int d^4x \mathcal{L}$ (after formally using the curve space-time) on the

symmetric metric tensor $g^{\mu\nu}$. The trace of the energy-momentum tensor (43) is not zero:

$$T_{\mu\mu}^B = 4\beta^2 \left(\frac{1 + \beta^{-2}S}{\mathcal{R}} - 1 \right). \quad (44)$$

According to [20], we define the modified dilatation current

$$D_\mu^B = x_\alpha T_{\mu\alpha}^B + V_\mu, \quad (45)$$

where the field-virial V_μ is given by

$$V_\mu = \Pi_{\alpha\beta} \left[\delta_{\alpha\mu} \delta_{\beta\rho} - (\Sigma_{\alpha\mu})_{\beta\rho} \right] A_\rho = 0. \quad (46)$$

As the field-virial vanishes in our case, the modified dilatation current (45) becomes $D_\mu^B = x_\alpha T_{\mu\alpha}^B$. Then the four-divergence of dilatation current is equal to the trace of the energy-momentum tensor (44):

$$\partial_\mu D_\mu^B = T_{\mu\mu}^B. \quad (47)$$

As a result, the dilatation (scale) symmetry is broken because of the presence of the parameter β with the dimension of the field strength. Thus, there is a difference compared to linear Maxwell electrodynamics: Maxwell equations are scale invariant. The conformal symmetry, which includes the one-parameter dilatation group of symmetry, is also broken [20].

6 Conclusion

We have formulated the generalized BI electrodynamics with two parameters β and γ . The model includes particular cases: BI electrodynamics if $\beta = \gamma$, linear Maxwell electrodynamics with quantum non-linear Heisenberg–Euler corrections, electrodynamics with one loop corrections due to vacuum polarization of arbitrary spin particles and others. We have obtained induced ellipticity in the background magnetic field due to the effect of vacuum birefringence. The upper bounds on the parameter $\Delta = 1/\beta^2 - 1/\gamma^2$ are obtained from the experimental data of BRST [16] and PVLAS [17] Collaborations. For the case of BI electrodynamics, phase velocities of different polarizations of the light beam are equal, and the effect of vacuum birefringence vanishes. It should be noted that the effect of vacuum birefringence is now of the experimental interest [16], [17], [21]. The canonical and symmetrical Belinfante

energy-momentum tensors, and the dilatation current have been obtained. It was demonstrated that the dilatation current is not conserved due to the non-zero trace of the energy-momentum tensor. Thus, the scale symmetry is broken because of the presence of the dimensional parameter β . If $\beta \neq \gamma$ the dual symmetry is also broken.

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